

**Coupled Mode Problems for Bottom Interacting Sound  
and  
Coupled Mode Problems for Bottom Interacting Sound:  
Student Support (Assert)**

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**LONG-TERM GOALS**

The long-term goal of this research is to improve our ability to model and predict VLF acoustic propagation in shallow water with particular emphasis on the range dependence of the medium and the geoacoustic properties of the bottom, and to quantify the various factors affecting the overall acoustic energy budget in shallow water propagation.

**OBJECTIVES**

Our scientific objectives are to incorporate the effects of sediment anisotropy, strong sediment attenuation, and the effects of both deterministic and stochastic medium properties into a local coupled mode propagation model, and to develop accurate theory and robust numerical algorithms for the shallow water propagation problem.

**APPROACH**

We are using an approach based on coupled local modes to carry out a systematic study of the effects of scattering, normal dispersion, anisotropy and intrinsic attenuation on a propagating shallow water acoustic signal with strong bottom interaction. The coupled mode theory is developed from the first order equations of motion for the stress and displacement rather than from the second order equations for a velocity or displacement potential. The later approach introduces coupling coefficients depending on the second-order derivatives with respect to the range coordinate of the local mode functions. These second-order coupling coefficients are an artifact of the formulation, and not present in the coupled mode theory based on the first order equations of motion.

**WORK COMPLETED**

We have quantified the effect of sediment anisotropy on mode coupling in a range dependent shallow water waveguide. Even weak non-transverse isotropy produces significant non-nearest neighbor inter-mode coupling. This has a significant effect of the distribution of energy in a shallow water waveguide, and also highlights the effect of acoustic energy coupling to quasi-SH in the sediments as a loss mechanism. This work has been submitted for publication (Soukup and Odom, 2001). The derivation of

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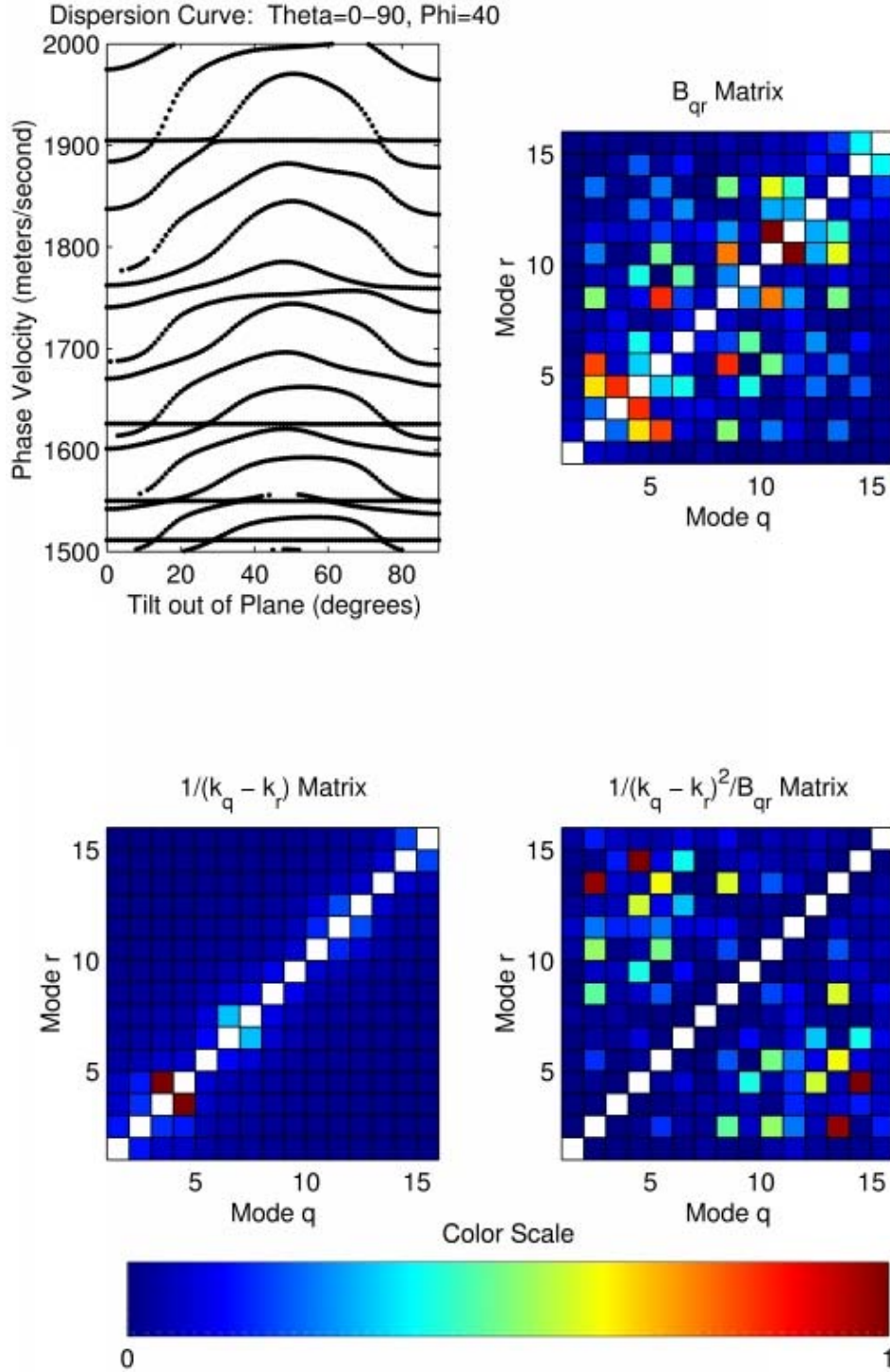
a number of technical results regarding the propagator is range dependent fluid–elastic waveguides was completed along with a Feynman diagram representation of the derivation of the Dyson and Bethe-Salpeter equations for the propagator in a range dependent fluid-elastic medium. This work has been submitted for publication (Park and Odom, 2001). Geophysics graduate student Darin Soukup, supported by the AASERT, derived theoretical results to incorporate the effects of volume perturbations in to the coupled mode code. He is currently coding this to test the theory.

## RESULTS

Strong range dependence in a shallow water waveguide will couple modes together, resulting in dispersion of a signal because of the differing group velocities of the various modes. In isotropic media such as fluids and isotropic elastic bottom material, and in transversely isotropic (TI) elastic media, the mode coupling is mostly dominated by the  $1/\Delta k$  term in the mode coupling matrix  $B_{qr}$ . The result is that the strongest interactions occur between nearest neighbors, and the coupling matrix is nearest neighbor dominant with the strongest interactions closest to the diagonal (Odom et al. 1996). If the bottom sediments are layered in such a way so that the normal to the layering is not also normal to the water surface, the symmetry of the TI medium is broken and the anisotropy of the bottom sediments takes on a more general form. The effects on a bottom interacting propagating acoustic signal can be significant.

Figure 1 illustrates the effect of rotating the symmetry axis away from the vertical of an initially TI medium. The upper left diagram in Figure 1 is a modal dispersion diagram. Modal phase velocity is plotted as a function of symmetry tilt axis for a model consisting of a 100m water layer over an anisotropic bottom sediment package. If the bottom sediments were isotropic, the lines across the figure would be perfectly straight. The modal phase velocities would be insensitive to direction, and the phase would be constant with respect to any angle. However, if the bottom sediments are anisotropic, modal phase velocities vary by as much as 50 m/s for this model. The frequency is 50 Hz. Shown in the upper right corner of Figure 1 is the coupling matrix  $B_{qr}$  for a symmetry axis inclination angle of  $20^\circ$ . Comparison of the matrix with the color bar on the bottom of the figure clearly indicates significant non-nearest neighbor inter-mode coupling. This has numerical as well as physical implications. It means that a larger number of modes must be retained when carrying out mode coupling computations, and that we cannot rely on just nearest neighbor coupling to capture the largest part of the interaction.

The two lower plots in Figure 1 separately show the contribution of the  $1/\Delta k$  term and the effect of the mode shape on the coupling matrix. The coupling matrix  $B_{qr}$  is a product of the  $1/\Delta k$  term and a second term that depends on the vertically integrated product of the individual mode functions. This second term contains all the information about the mode shapes. The lower left matrix is just the  $1/\Delta k$  matrix and is of course nearest neighbor dominant. The strongest coupling is between the mode pair (2,3). The matrix in the lower right corner is the ratio of the  $1/\Delta k$  term and the second term comprising the matrix  $B_{qr}$ , which depends on the mode shape. If this ratio is large then the  $1/\Delta k$  term is dominant. If the ratio is small, the actual mode shape makes the greatest contribution to the coupling. This plot clearly shows that there can be very strong non-nearest neighbor coupling. The redder values, corresponding to stronger coupling, are distributed throughout the matrix far from the diagonal. This result is somewhat counterintuitive, because a cursory evaluation of the problem would lead one to assume that the  $1/\Delta k$  term should dominate. An additional implication of this work is that the acoustic modes may couple to a quasi-SH mode in the sediments. Because the SH is so strongly damped in the sediments, ignoring this coupling to the quasi-SH would result in underestimating the total loss experienced by the acoustic signal. This work has been submitted for publication (Soukup and Odom, 2001).



**Figure 1.** This figure shows a dispersion curve (upper left corner), complete coupling matrix (upper right), a  $1/\Delta k$  matrix (lower left), and the ratio of the  $1/\Delta k$  term and the mode shape factor of the  $B_{qr}$  matrix. The model is a 100m water layer over anisotropic sediments. The frequency is 50 Hz. The dispersion curve shows the variation of modal phase velocity with the symmetry axis inclination angle of the sediments. The coupling matrices show that the coupling in a shallow water waveguide can be dominated by non-nearest neighbor interactions among the modes. The matrix in the lower right corner makes it clear that the mode shape itself can play a dominant role in the coupling, a rather counterintuitive result.

The propagator for a range dependent can be cast into a number of different forms, each of which emphasizes particular physical aspects of the propagation. The starting point for the derivation is the product integral representation of the propagator (Gilbert and Backus, 1966). One representation of the propagator has the form of a reflectance operator with terms that correspond to the lateral modal impedance of the medium. Another representation is a “product of a product” of two evolution operators, one of which represents the phase integral of each modal wavenumber within a range sub-interval, and the other which denotes the transition from one mode to another within the same spatial sub-interval. This representation reduces to the WKB approximation in the limit of vanishing inter-mode coupling. Yet another representation takes the form of a phase space path integral. This work along with a Feynman diagram representation for the derivation of the Dyson and Bethe-Salpeter equations for a range dependent fluid-elastic medium with random heterogeneities. This work has been submitted for publication (Park and Odom, 2001).

## **IMPACT/APPLICATIONS**

Highlighting the importance of sediment anisotropy in mode coupling and signal loss is an important step in the understanding of acoustic propagation in complicated heterogeneous waveguides. This research is directly applicable to predicting the effect of a complicated shallow water environment on the acoustic field.

An important application to another field of the theoretical results we have derived is the generation of oceanic T-waves. The modal scattering theory of Park and Odom (1999) was used to show that modal scattering from a sloping bottom or seabed roughness can excite the low order acoustic modes known to carry the T-waves (Park et al., 2001). This application was funded under the National Ocean Partnership Program (NOPP).

## **TRANSITIONS**

Modal methods for modeling in random range dependent shallow water waveguides should provide important constraints on the most significant waveguide properties affecting propagation at low frequencies.

## **RELATED PROJECTS**

Our research is directly related to other programs studying surface, volume and bottom interaction effects, including 6.2 and 6.3 efforts to quantify bottom backscatter and bottom loss effects in littoral regions.

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